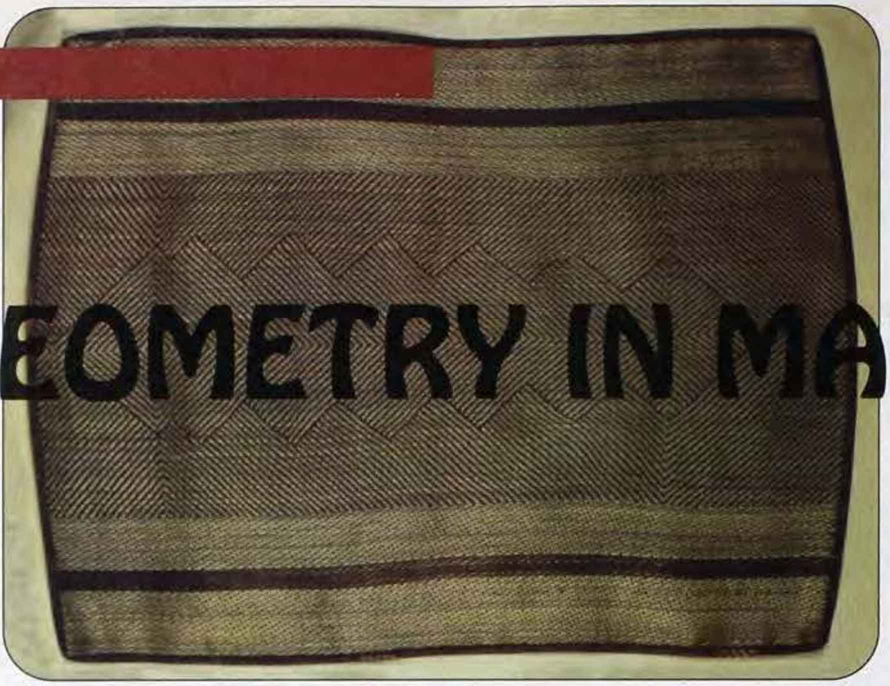
Ron Eglash



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GEOMETRY IN MANGBETU DESIGN

The author extends thanks to Michael Robertson for his geometry advice and to Enid Schildkrout for her anthropological expertise. The author would appreciate hearing readers' ideas or experiences with instructional application of this material.

One difficulty in the use of African designs for mathematics instruction is our lack of knowledge of the artisans' concepts and techniques. In many cases, that knowledge has been lost because of modern disruptions of traditional societies. Although the Mangbetu also suffered such losses—their modern descendants no longer create many of the patterns we discuss—their preoccupation with a geometric approach to design was so strong that some of its basic elements can be discerned in artifacts that have survived. This article introduces a few examples of Mangbetu designs and examines their underlying structure. We hope that teachers and students will join us in discovering the geometric basis for these beautiful patterns.

Originally, the most important aesthetic was abstract geometric design

WHO ARE THE MANGBETU?

The Mangbetu occupy the Uele River area in northeastern Democratic Republic of Congo. Archaeological evidence reveals iron smelting in the area since 2300 B.C., but the Mangbetu, who came from drier lands around present-day Uganda, did not arrive until about A.D. 1000. Through conflict and cooperation, they exchanged cultural traditions with other societies of the area: Bantu-speaking peoples such as the Buda, Bua, and Lese; and Ubangian-speaking peoples such as the Azande, Bangba, and Barambo. Around 1800, a number of small chiefdoms were consolidated into the first Mangbetu kingdom. Although this kingdom lasted only two generations, a tradition of courtly prestige continued even in small villages and spread to many of the Mangbetu's trading partners. This combination of cultural diversity, exchange, and prestige resulted in a thriving artistic tradition.

African Reflections: Art from Northeastern Zaire (Schildkrout and Keim 1990) contains a detailed account of Mangbetu history and traditions. Schildkrout and Keim begin their analysis by showing that the most famous aspect of Mangbetu art, the "naturalistic look," was rare in the traditional Mangbetu society of the nineteenth century. During a research expedition to the Congo in 1914, when the photos used here were taken, mammalogist Herbert Lang became fascinated with lifelike carvings of human figures, and as word spread that he was paying high prices for them, the Mangbetu produced more of these carvings. Other collectors came to buy these pieces, and eventually the economic rewards for producing naturalistic Mangbetu art became so strong that it replaced other styles.

This history puts the presumed emphasis on the concrete and natural into perspective. Schildkrout and Keim show that originally the most important aesthetic was not naturalism but abstract geometric design. The indigenous fascination with artifice and abstraction was ignored, however, and Western conceptions of Africans as nature-loving "children of the forest" became a self-fulfilling expectation.

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But the artifacts and photographic records from the 1914 expedition provide us with excellent examples of traditional Mangbetu patterns, as well as an opportunity to infer some Mangbetu techniques.

MULTIPLES OF THE 45-DEGREE ANGLE: A MANGBETU DESIGN THEME

As seen throughout Schildkrout and Keim's book, several design techniques might be associated with Mangbetu patterns. Ceremonial knives, for example, often show what appear to be precise exponential curves, and baskets frequently use hexagonal tilings. But the most compelling visual theme in the designs is the persistent use of multiples of the 45-degree angle.

This simple basis for design can result in surprisingly complex patterns, as shown by the decorative end of an ivory hatpin in **figure 1.** The carving shows three interesting structural features **(fig. 2).** First, each head is larger than the one above it and faces in the opposite direction. Second, each head is framed by two lines, one formed by the jaw and one formed by the hair; these lines intersect at approximately 90 degrees. Third, an asymmetry is evident: the left side shows a distinct angle about 20 degrees from the vertical.

Fig. 1
Mangbetu ivory sculpture

Fig. 2
Geometric features of the ivory sculpture

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The structure shown in **figure 3** accounts for all these features. One can use any iterative process to construct this sequence of shrinking squares, bisecting one square to create the length of the side for the next square, as indicated in the diagram. We will never know for certain if this iterative-squares construction was the basis for the sculpture's design, but it does match the features identified in the foregoing. In the ivory sculpture, the left side is about 20 degrees from the vertical. In the iterative-squares structure, the left side is about 18 degrees from the vertical, as **figure 4** indicates. The iterative-squares structure also illustrates

Fig. 3
Geometric analysis of the ivory sculpture

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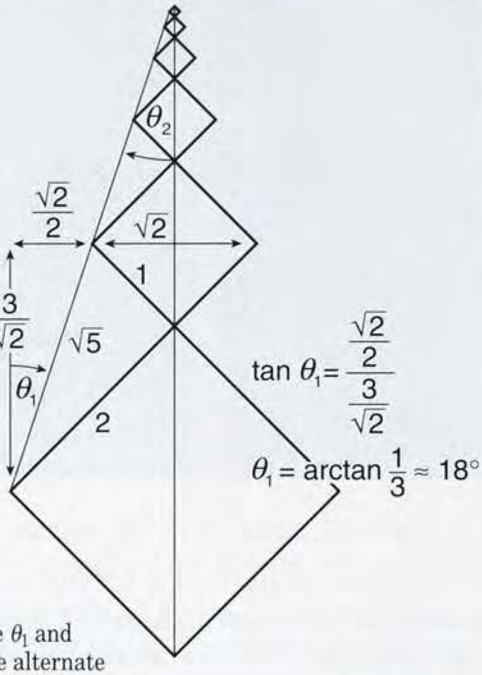
what fractal geometers would call the ivory sculpture's *scaling symmetry,* that is, its shape-preserving transformations in size (cf. Wahl [1995, 156–57]). Designs with scaling symmetry, or "self-similarity," are found in many other African societies (Eglash 1995a, 1995b, 1997).

Designs with scaling symmetry are found in many African societies



Since θ_1 and θ_2 are alternate interior angles of a transversal intersecting two parallel lines, $\theta_1 = \theta_2$.

Fig. 4
Geometric relations in the iterative-squares structure

Simply asking students to attempt to discover the geometric structure of the sculpture in **figure 1** might be a worthwhile investigation, since they could employ a variety of techniques, including tracing on graph paper, ruler-and-compass construction, and computer-drawing programs. Even if they do not discover the iterative-squares structure, students may be more motivated to explore its mathematical properties after the search. The infinite construction produces a number of interesting patterns, such as the numerical sequences produced by measuring sides, diagonals, or areas of successive squares, and the trigonometric relations that result, for example, derivation of the value arctan 1/3. It can also be used as an example for computer-programming techniques, since the iterative nature of the construction lends itself to a recursive procedure, as often used in LOGO, L-system graphics, and other drawing languages (cf. Saupe [1988]).

BODY GEOMETRY

If the iterative-squares structure in **figure 3** really is the underlying design principle for the ivory sculpture, what was its origin? One possibility is the Mangbetu's geometric style of personal adornment. **Figure 5a** shows a Mangbetu hair style, popular during the time that this carving was created (about 1914), which featured a disk angled to the vertical at 45 degrees. Men often wore a hat with the top flattened to form the same angle, as **figure 5b** shows. Just as a plane cuts diagonally through the top of the heads in the ivory sculpture of **figure 1,** real Mangbetu headdresses also terminated in a 45-degree angle.

In **figure 5b**, the Mangbetu chief's hat has an ivory hatpin, ending in a disk perpendicular to it, inserted perpendicular to the hat. To its right, a small ivory arrow pinned to the hat points horizontally, forming an angle of 135 degrees with the hatpin. Each part of the ensemble was aligned by a multiple of the 45-degree angle. This adornment

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(a)
Mangbetu woman weaving headband

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(b)
Mangbetu chief

Fig. 5
Geometric design in Mangbetu personal adornment

style included artificial elongation of the head, which is clearly visible in the photograph in **figure 5b.** Elongation was accomplished by wrapping a cloth band around the head of an infant; the woman in **figure 5a** is weaving one of these bands. Head elongation resulted in an angle of 135 degrees between the back of the head and the neck.

MANGBETU MUSICAL INSTRUMENTS

Although the Mangbetu geometric conception of the body may have inspired the 45-degree-angle-design theme, those designs were certainly not limited to simple mimicry of anatomy, as we can see in Mangbetu musical instruments. The upper surface of the drum in **figure 6a,** for example, is cut at a 45-degree angle to the vertical, but it is not anthropomorphic. The instrument shown in **figure 6b** has a resonator that meets the vertical tuning stem at a 135-degree angle. Furthermore, even in the case of anthropomorphic designs, the artisans elaborated on the human form in ways that show creative, and not merely imitative, applications of geometrical thinking. For example, although the end of the tuning stem shown in **figure 6b** has an anthropomorphic decorative motif, these human heads are not simply mimicking human form. In **figure 5b** we saw that the Mangbetu had a 135-degree angle between the back of the head and the neck. The carved heads in **figure 6b** have a 90-degree angle between the back of the head and the neck. Such distortions indicate active geometric thinking rather than passive reflection of natural anatomical angles, which, recalling the artificial head elongation, were actually not so natural.

ABSTRACT MANGBETU DESIGN

Finally, abstract Mangbetu designs make use of multiples of 45 degrees. The woven mat shown in **figure 7,** for example, has overlapping squares reminiscent of the iterative structure in **figure 3.** Perhaps the most complex example is the mural in **figure 8a,** which shows a tiling pattern composed of triangles and parallelograms. The relations of side, base, and height of these obtuse triangles suggest construction by 3-4-5 right triangles **(figs. 8b** and **8c).** It may be that the artist combined 3-4-5 triangles in pairs to obscure the original design method, leaving an intuitive impression of geometric precision without visual clues about its underlying structure.

Can the 3-4-5 triangle be constructed using the Mangbetu's technique of combining multiples of the 45-degree angle? It is readily derived from a Mangbetu design that we have already examined: the iterative-squares structure of **figure 3. Figure 9** shows the fit between the two figures in schematic form, and **figure 10** provides the geometric derivation of these relations.

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(a)
Drum

The original of this photo is in the Rietberg Museum Zurich, Eduard von der Heydt collection.
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(b)
Harp

Fig. 6
Geometric design in Mangbetu musical instruments

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Fig. 7
Mangbetu woven mat

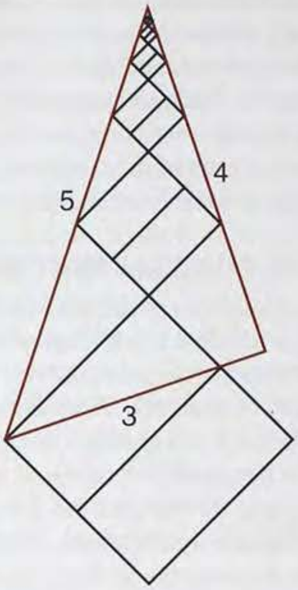

Fig. 9
Schematic fit of 3-4-5 triangle to the iterative-squares structure

It is not clear how the original artisans conceived of or interrelated the geometric structures that we have reviewed—the relation between **figure 3** and **figure 8,** for example, may be much less direct than shown in **figure 9**—but collectively they show an intentional use of geometric design and prompt many further questions. Was special terminology used to distinguish these angles? Did the Mangbetu discuss relationships among these figures or even formulations equivalent to Euclidean axioms? Unfortunately, these questions were never asked;

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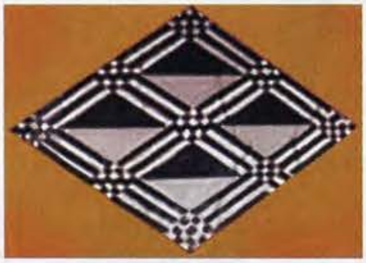
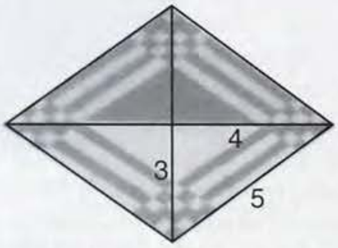
(a)
Mural



(b)
Mural close-up



(c) Geometric analysis of the mural close-up

Fig. 8
Geometric design in Mangbetu mural

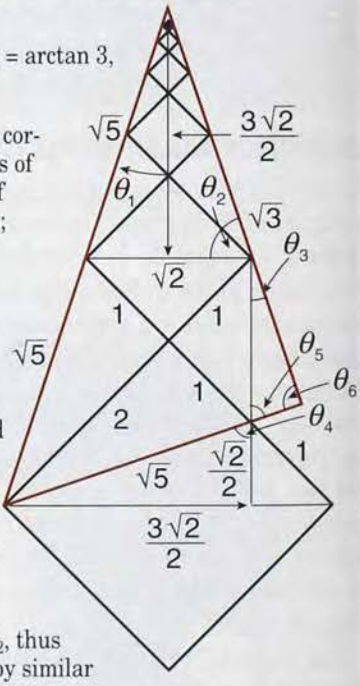
Given the previous results from **figure 4** and the symmetry of the structure, $\theta_1 = \arctan 1/3$,

$$\tan \theta_2 = \frac{\frac{3\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}},\ \theta_2 = \arctan 3,$$

and θ_3 and θ_1 are corresponding angles of the transversal of two parallel lines; thus $\theta_3 = \theta_1 = \arctan 1/3$.

$$\tan \theta_4 = \frac{\frac{3\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}},$$

$\theta_4 = \arctan 3$, and θ_4 and θ_5 are opposite angles of the intersection of two lines; thus $\theta_5 = \theta_4 = \arctan 3$.



$\theta_3 = \theta_1$ and $\theta_5 = \theta_2$, thus they are formed by similar triangles. Therefore,

$$\theta_6 = 90^\circ, \quad a = \frac{1}{\sqrt{5}}, \quad b = \frac{3}{\sqrt{5}}.$$

Fig. 10
Derivation of the 3-4-5 triangle relationship from the iterative-squares structure

anthropological theories of the time emphasized functionalist perspectives, concerned more with the social roles of beliefs than their internal content.

We do know, however, that the Mangbetu did not use any of their sculptures as religious icons, and Schildkrout and Keim suggest that the few symbolic interpretations offered to Lang were most likely fabricated on the spot. Modern informants (Schildkrout and Keim 1990, 100) report that the creation of a design reflected only the artisan's desire to "make it beautiful and show the intelligence of the creator." This statement suggests another reason for artisans to adhere to angles that are multiples of 45 degrees: if no rules existed, then it would have been difficult to compare designs and demonstrate ingenuity. By restricting the permissible angles to a small set, the Mangbetu could better display their geometric accomplishments.

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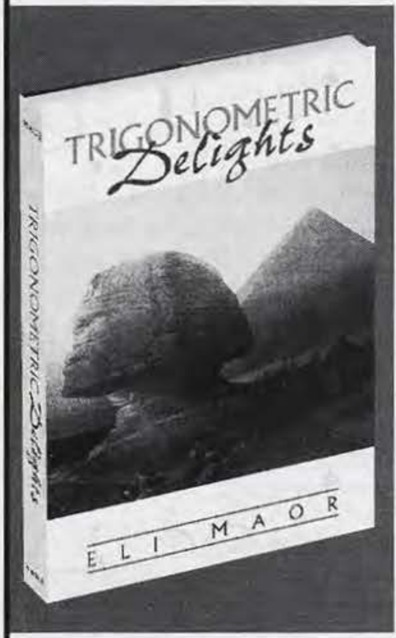

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