Culturally-Relevant Algebra Teaching: The Case of African Drumming

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Abstract

Aside from being its own field of study, algebra has long been a servant to upper division courses and arguably to productive citizenship. Algebra plays a key role in courses like chemistry and trigonometry, which are the kinds of upper level courses required of high-tech, usually high-paying, occupations such as engineering. Keeping learners out of upper division courses also means limiting their potential for active citizenship in society and economic success (Povey, 2003). Many (see Burton, 2003) argue more philosophically that access to and knowledge of mathematics leads to an empowered populace, making it a key to social justice. Unfortunately, algebra can appear to be a tool of injustice when it protects upper division mathematics courses from potential students. Intentions are understandable since success in those courses requires algebraic thinking. Under-prepared students often fail. But, the net result has been to preclude students from those courses rather than to ensure students are prepared for those courses. Robert Moses (Moses & Cobb, 2001) suggests that algebra must move beyond its role as a pre-requisite hurdle. He refers to algebra as a *civil right*, suggesting that all learners have the *right* to know algebra. Viewing algebra in this manner elevates its justice status, allowing all students greater opportunity to economic success.

If such equity is a desirable goal, the mathematics education community should develop teaching strategies that resonate with all learners. For algebra teachers to provide teaching and learning opportunities that promote access for more students, we need insightful pedagogical knowledge and deep algebra knowledge. Practicing culturally relevant pedagogy is an approach that may require teachers to develop these kinds of knowledge. By posing problems that are relevant (Hart, 2003) and that allow students to respect, appreciate and/or celebrate other cultures (Boyer, 1990) teachers' pedagogical knowledge may become stronger. Driscoll (1999) suggests that a useful strategy to support efforts to deepen algebra knowledge is to provide teachers with experiences learning algebra in new ways and in unexpected settings. Together these kinds of

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knowledge impact teachers practice. In this paper, we describe an atypical context, African/Afro-Cuban drumming that has the potential to connect teachers' knowledge about pedagogy and algebra by stimulating algebra learning.

Teacher knowledge

Deep content knowledge is significant because it frees teachers from having to think about their own mathematics knowledge and instead "to attend to the mathematics in what children say and do" (Schifter, 2001, p. 71). Teachers who know mathematics deeply have forged many conceptual connections, both within and outside of mathematics. Ma and Kessel (2001) put forward the notion that teachers' mathematical knowledge is linked to pedagogical knowledge in ways that could not and should not be separated. They note that *profound* mathematical knowledge manifests itself in a teacher's ability to "reveal and represent ideas and connections in terms of mathematics teaching and learning" (p. 12). When teachers develop profound knowledge, they understand something about the nature of learning that allows them to engage with students, recognizing unique ways of thinking about a problem. That is, teachers more acutely understand thinking and the learning process (Ma, 1999).

Much of the weight of learning about mathematical connections should be carried by mathematics teacher educators first, so it can be transferred to teachers. Driscoll (1999) provides insights about content for professional development in the specific case of algebraic thinking and learning. When teachers discover algebraic connections in what he calls "unexpected" places, their pedagogy is invigorated. When students experience algebra lessons in unusual settings, they are encouraged to develop algebraic habits of mind. Creating innovative lessons requires teachers to rethink their understanding of algebra and embrace goals and methods that differ from traditional presentations. This is a somewhat recursive experience. As teachers explore algebraic ideas in atypical settings, they gain new insights about how to provide this sort of instruction. And, as Ma & Kessel (2001) noted, pedagogy and content become/are intertwined.

Culturally-Relevant Algebra Teaching and Content

The premise here is that by adapting existing curricula to tap into the different knowledge bases of all students, we provide arenas for studying algebra in unexpected contexts, thus providing more opportunity for success in the critical algebra pre-requisite. This suggestion is a tall order. But, it is in line with theory about how students learn mathematics. Pirie & Kieren (1994) suggest that mathematical learning must begin with personal knowledge in order to invest the student in the process of learning. Most algebra-based textbooks include a variety of word problems with a goal of providing learners with real-world situations. Many contexts are restricted to the expected, scientific situations like paddling up and down rivers, two trains meeting on railroad tracks, finding areas of rectangular gardens, or mixing chemical solutions. Although these are situations in which algebra should be employed, there are additional contexts that require rigorous algebraic thinking and learning. Cultural context have the added benefit of drawing from a more diverse collection of learners' personal knowledge bases.

As teachers, we tend to teach like we were taught (Lortie, 1975; Russell, 1997; Schifter, 1997). If we learned in classrooms where teachers literally followed traditional textbooks, it would be no surprise that we teach algebra using the same kinds of *expected* contexts –such as paddling up and down a river. To create new, innovative lessons from culturally-relevant contexts, teachers will need to expand their experiences and knowledge about algebra or teacher educators will have to provide such experiences. That is, we all must learn to "be alert to finding algebraic thinking in ostensibly nonalgebraic settings" (Driscoll, 1999, p. 21).

Algebraic thinking in these new contexts must retain rigorous mathematics. The tenor of the lessons must be coherent and purposeful. A short activity on weaving or pottery, is not enough to promote algebra to a tool for social justice (Weist, 2001). To develop and present a robust collection of lessons that honors a culture, teachers must know about cultural connections to mathematical thinking and related content. Addressing culturally-relevant connections in a classroom requires teachers to study and understand a culture before they can respect and celebrate it mathematically. Once mathematics teachers gain new connections to mathematical knowledge, because of their study of another culture, passing a celebratory attitude to their students may become more natural. As mathematics knowledge becomes visibly intertwined with cultural knowledge, rather than isolated from it, teachers forge many cultural connections to the mathematics and they will be more aware of the potential for expanded knowledge among their students (Gerdes, 1998). Ma (1999) refers to this kind of knowledge as *thoroughness* of mathematical knowledge. Hence, we describe the potential of studying some of the history of African drumming.

The cultural component: African/Afro-Cuban Drumming

The creation of a mathematically robust lesson with a meaningful cultural background requires a thick understanding of the culture as well as the mathematics. To begin to understand a culture means qualitatively understanding the roles of both typical members of the society and of the mores and codes established by the leaders of the society. This kind of knowledge can provide some measure of understanding that moves beyond superficial knowledge of a culture and to a more connected, respectful understanding. Therefore, before explaining the mathematics component, we delve slightly further into the culture of African drumming.

Many things help to define a culture. One of the most obvious things is a community-wide event. Things like county fairs, art festivals, or school plays can give a community some identity and provide opportunities to interact. Such defining events are found in all cultures and can be a way for an outsider to gain some understanding of a culture. The events of interest here are the African drumming and dancing ceremonies that trace their roots to many ancient African cultures. While we run the risk of incorrectly implying that it is appropriate to group all African ceremonies together, there are common threads among celebrations from many African cultures: the importance of the drums, drummers, dancers, and village leaders/elders.

The Burundi

In Burundi, drums are sacred and represent, along with the king, the powers of fertility and regeneration. Along with the large ingoma drums made of hollow tree trunks covered with skin, the amashako drums provide a continuous beat, and ibishikiso drums follow the rhythm of the central inkiranya drum. One of the yearly ceremonies of the Burundi people was the Mwami ceremony, during which special drums were used. The drums had their own special hut for storage and took the entire year leading up to the ceremony to be properly prepared (Grund, 1985). The purpose of the Mwami ceremony was to honor the king and his ancestors. These ancient traditions have been preserved and are on display worldwide as the Royal Drummers of Burundi, the former royal percussionists, perform worldwide, demonstrating rhythms passed down from father to son through "the oral tradition" of African drumming (indeed, African culture).

Drummers (members of the culture)

Only master drummers (Batimbo) were allowed to play the drums because only Batimbo had the proper life-long training to play the drums for the month-long ceremony. For a ceremony to be successful, it had to include dramatic dancing and intense fluctuations in heart rates. So, the master drummers had to learn not only to play the music for the ceremony, but also to learn when and how to introduce unplanned rhythms into the music. By reading their audience, ritual drummers, like the Batimbo, learned to improvise the rhythms of the traditional drum music to sway the crowd and the dancers (Charreaux, 1992; Locke, 1987). Professional drummers believe that the human heart is one of many rhythms at work in the human body (Anderson, 1994; Cohen & McFall 1994). The Batimbo were trained to recognize the cumulative pulse of their audience, lock into it, and adjust the music in ways that naturally swayed them (Steed, personal communication, 1984).

Due to the importance of this skill and the importance of the drums, drummers held a noble ranking in the culture. However, this noble ranking could be short-lived for some drummers because the Mwami ceremony was a month-long ceremony and drummers have been known to die from exhaustion at the conclusion of the ceremony. The noble ranking also afforded the drummers the honor of playing the drums for a few days following the ceremony before the drums underwent the year-long preparation for the next year's ceremony (Grund, 1985).

The Mathematical Knowledge of the Drummers

Locking into the crowd's cumulative heartbeat requires mathematical knowledge, but traditionally, drummers weren't taught to view what they were doing as being connected to math. The drummers must catch the brief moment in time when everyone's heartbeat is in synch and then find a way to control that moment and move quickly away from that point. So, how does a drummer use mathematics to do that? Mr. Stevens responds below. [Editors Note: We chose to italicize Mr. Stevens words for emphasis.]

> *I will have to forego the scientific tradition of source referencing information to explain what happens as I drum and teach math; this is because drumming isn't a science in the African/Afro-Cuban tradition, and, much of what is known about African and Afro-Cuban drumming is from word of mouth histories. This is our culture. This is how I learned it and that is part of the beauty of the African drumming tradition; in its purest form it requires connectedness to the source. It cannot be fully understood from writings alone. It is earthy, breathing, and it is alive. When you drum, when you dance to the drum, you must be connected or you will miss the most important experiences.*

Thus, what I've learned, what I know, has come from the source, from elders within the tradition and from those who have studied with and lived among those elders.

I feel as if I must defend the tradition, concerned that it may sound like a bit much to take in for the inexperienced. But, this is how we operate: It isn't science in our hearts it is spirit in our hearts. We feel. Asking us how we do what we do is akin to asking the Shaman where "knowing" comes from. It's like asking an empathic where feeling comes from. It's just there and it's always been there. I was supposed to do this. I'm born to do this. My elders did this and passed it down to me. It's always been there with me. It's like a 6^{th} sense.

The "it" I speak of is an innate knowing of rhythm. And, because rhythm is central to all life and living things we can come to know the state of a person through it. The master drummer understands that intuitively. We are taught to find unity in the crowd we play to. That unity will reflect the contents of the listener's heart. The heart that is clouded will not respond to rhythm in the way that a clear heart responds. The clouded heart resists the unity called for by rhythm and meter. If one wants to see an example of this, watch children when they are in the presence of rhythm. They move instantly and (typically) vigorously. They unite with the rhythm, the message. Their hearts are clear. Watch any concert; sooner or later a foot will tap, a head will nod; that person at that time has responded as evidenced by their movement. Their heart wants to unify with the rhythm and their body tells us so through their movements. This is very natural.

When we play to you we will eventually see your true intentions. You will reveal what is in your innermost being. The doubtful heart moves awkwardly, almost shamefully, with reservation, but it will move (especially when others are moving with it). Anger and rage, hurt and loss, joy and peace are all made visible by our rhythm. We know you well shortly after we begin playing, and we will, if we are attempting to move you, adjust our cadence until we do exactly that.

The above point illustrates the core of the African drumming tradition; we ultimately are concerned more with the heart of a person than with a person's doings. Transform the heart and the rest will take care of itself. The drum can do that. The drum can speak. The drum can heal.

The marriage of drumming and math is to me most special because the elders of the tradition, I would argue, would never have been concerned with this connection. Drumming for them was likely far more serious an endeavor than education or mathematics. They played for birth, for regeneration, for the hearts of their villages, kings, and nations. They played for their gods. As we fast forward from say 13^{th} century Mali to the present, we have become inundated with the products of the scientific movement, the technological movement, the depersonalization movement. Product is now king and students suffer because they lack connectedness to their subjects, their teachers, to each other. The drum does some of its best work here. When I enter a classroom and begin to play I take the students to the most elemental common denominator, movement via rhythm. It's ok to move again, to feel again; and they're

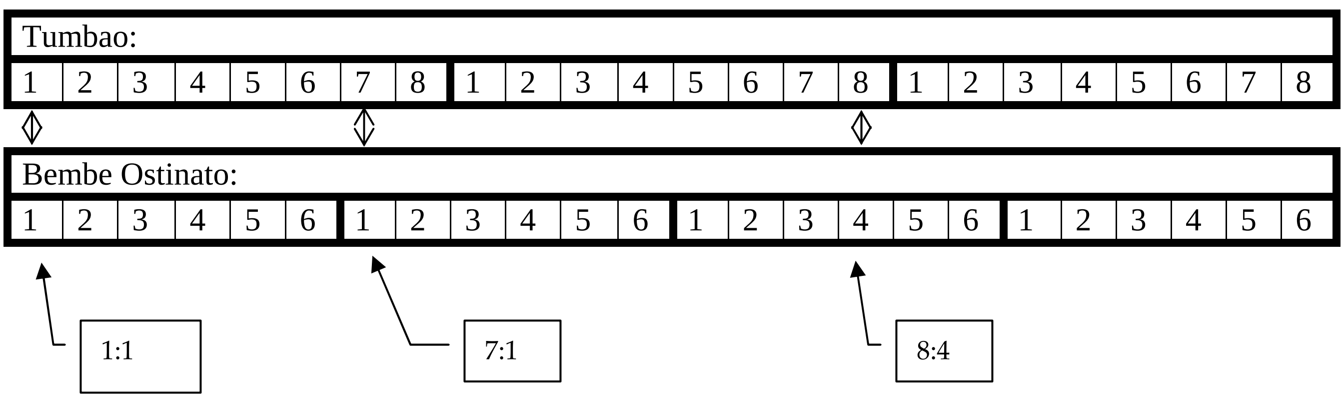
> *feeling math - ratios, fractions, polyrhythms. That's math. The subdivisions are apparent as their hearts, feet, and my drums communicate. You can hear it, you can see it, and you can feel it. It's like the ancient Nigerian tradition of playing a rhythm directly on the body of the student. The student doesn't just watch the hands of the teacher and try to mimic their movements, they feel the pattern. It is internalized. So it is with math and students once they've felt a ratio or a fraction via my drum. It's a whole body type of learning. It's not cerebral it's guttural; natural; it's easy. That's the "system".*
>
> *Once the student has experienced rhythm they've experienced math. They just don't know it yet. I simply explain what they have experienced in a language we call mathematics…I could have explained it in Swahili instead…math is just another language to me. And languages are important to our oral tradition. In fact, we are trained to vocalize the rhythms we play, to sing them, as we play them. There are entire systems of vocalizations for African drumming. Those systems preserved the rhythms when Africans were enslaved and their drums were taken from them as they were sent throughout the world. So, once a child has felt math there's not much left but the vocalization. This is our culture. And it lends itself perfectly to the teaching of mathematics.*

African drumming is a collection of rhythms that are played simultaneously. Just as "polygon" is a term for a geometric shape that is many-sided, "polyrhythm" is a term used to describe music that is made from simultaneously playing many rhythms. These repetitive rhythms are of different lengths and bring different vibrations into play, based on how the drummer strikes the drumhead or the kind of drum used. The drummers may have at their disposal rhythms composed of several different counts, for example, 8 counts, 6 counts, or even 12 counts. They decide which rhythms to play and when to play them based on their read of the audience. In essence, the moment at which the participants' heartbeats are in synch occurs when these different rhythms are aligned. Then, the drummer incorporates the rhythms of different lengths to bring the audience into the music. By analyzing this process algebraically, we can gain new insights into the mathematical thinking of the drummers.

Of course, many rhythms make up African drumming and many drummers would be playing many rhythms at any given moment. For simplicity, in this paper, we analyze only two rhythms: the tumbao (toom-BAH-oh) and the bembe ostinato (BEM-bay aus-tin-AH-toe). The tumbao is an 8 count rhythm and the bembe ostinato is a 6 count rhythm. Analyzing the result of playing these two rhythms simultaneously allows us to understand how the drummer's awareness of and use of mathematics caused the king, dancers, and on-lookers to interact with the songs. The tumbao is a melodic rhythm that incorporates several sounds and vibrations. It is typically played on the tumbadora, a large conga drum (Cohen, Cohen, & Fedele, 1992; Bossierie & Panicka, 1992). The bembe ostinato is a short repetitive rhythm, much less diverse than the tumbao. It incorporates one kind of vibration and tends to be a constant underscore for more diverse rhythms.

Assuming both individual rhythms are played simultaneously, there are moments when they both begin on the same count of "one". Ritual drummers are trained to recognize when the aligned count of "one" matches the dancers' heartbeats. At this moment in the music is the count of the polyrhythm is 1, but the difference between the counts of the individual rhythms is zero. The drummers recognize this moment, and move through the rhythms, which are of different length, in order to fall away from the syncopated heartbeats. For instance, when the tumbao rhythm is on count #7, the bembe ostinato has returned to count #1. So, at that moment, the difference between the individual rhythms (7 – 1) jumps to 6. As dancers react to this drastic change, they experience a kind of rhythmic dissonance that needs to be physically resolved by their bodies. See Figure 1 for some examples of the counts of the polyrhythm and the two individual rhythms being analyzed. Resolving the dissonance results in dramatic swaying and keenly-accentuated dance movements, which are deeply appreciated by the king. The drummers move cyclically through their rhythms, sometimes with the rhythms aligned, when the dissonance is resolved, and sometimes with the rhythms misaligned, when the dissonance is at its height. Each time the rhythms are aligned, the dancers and listeners are brought back into the music. In this manner, the drummer controls the ceremony, which explains their importance in the culture. However, experienced dancers can also silently communicate a desire to maintain the dissonance. In those cases, the drummers introduce flairs and other off-cadence beats that appear to “trick” the audience into thinking that the dissonance is about to resolved but that send the rhythms off into different directions. Thus, maintain the heightened adrenaline and exaggerated dancing moves.

Figure 1 – Counting the Beats of the Two Rhythms



The algebraic component: Functions

Studying the rhythmic dissonance in the different African drumming rhythms within a polyrhythm may be accomplished with algebra. The dissonance may be literally described as the difference between the numerical values of the individual rhythms. By comparing the “counts” of competing rhythms making up the polyrhythm, interesting patterns are formed. Patterns often provide a background for algebraic thinking. Beginning in Kindergarten, children develop algebraic thinking when they copy, extend, or create patterns (National Council of Teachers of Mathematics, 2000). The patterns in the rhythmic dissonance of African drumming can become a natural place to connect algebra to prior patterning knowledge (Stevens, Sharp, & Nelson, 2001).

When the differences of the competing African Drum rhythms are studied, a mathematical structure may be imposed upon the polyrhythm.

Lesson Set - up

The events described here occurred in a summer session, teacher education graduate course in algebra. The algebra teachers hailed from different locations across the United State (e.g., Alaska, Montana, Virginia, Texas and Arizona.) Many of these teachers were pursuing a master’s degree. All had taught algebra at least once. As the teachers flowed into the classroom, they were exposed to an on-going video clip of a conga player playing the Tumbao and Bembe Ostinato rhythms simultaneously (Center for Technology in Learning and Teaching, Curry School of Education, & Center for Learning and Teaching in the West, 2006). Naturally the conga player introduced his own flairs into the polyrhythm. Discussion of the mathematical ramifications of his flairs goes well beyond the scope of an initial lesson and furthermore discussion of the flairs is unnecessary to experience the algebra of the polyrhythms. After a few minutes of listening to the rhythms, the teachers then viewed a corresponding video clip of a dancer interacting with the same music.

Discussion questions

The following questions were offered for discussion:

(1) Do you notice any consistent patterns in the dancer’s movements?
(2) What do you feel while listening to the music?
(3) What sorts of emotions do you think the dancer is experiencing as she dances to the music?
(4) Why do you think the drummer has his eyes closed as he plays the drums?
(5) Watch his hands. Do you notice any differences in the way his hands strike the drumhead?

Their answers were used to weave the cultural information about the Burundi (described previously) into the teachers’ knowledge bases. They noticed how the drummer’s eyes were closed. Our drummer (and co-author, Stevens) says he sees and hears rhythms in his head, as though they exist in three-dimensional space. He imagines vectors projecting along various planes. He feels as though he's in this space and is communicating for the vectors. "The vectors are of differing intensities and lengths. When they are aligned in their distances the rhythm has unified at "the one", the point where the polyrhythm resolves and all players are at the beginning of their patterns together,." says Stevens. This provided the impetus for a brief lecture about the solemn nature of the drummer's role in the African culture to which the drummer in the video felt connected. The teachers noticed the varying intensity of the dancer's movements. This allowed for discussion of the importance of dancing and a brief description of the celebration dance from which the dancer was drawing many of the free-form movements she exhibited. It also allowed us to segue to the need to analyze the numerical differences involved in the music, for it is those differences within the polyrhythm that cause the dancer's movements to vary.

Differences & Patterning

A good way to analyze the mathematics of these rhythms is to first symbolically represent the rhythms. The teachers were introduced to the following symbols developed by Dworksy & Sansby (1994) as a way to represent hand strokes. Four hand strokes were used by the drummer: Open tone (○), Slap (▲), Heel & Toe (◒ & ●) and a rest (~). (note: The Heel & Toe generally appear together as a back and forth movement and so are represented here together. However, they are considered separate hand strokes.) The 8-count tumbao can be represented as ◒ ● ▲ ● ◒ ● ○ ○. The 6-count bembe ostinato can be represented as ○ ○ ~ ○ ○ ~. Although some might argue that the rhythm is a 3-count rhythm, it is important to think of it as a 6-count rhythm since that is the manner in which drummers and dancers feel the rhythm. Together the two rhythms look something like Figure 2.

Figure 2 – The Tumbao Rhythm and the Bembe Ostinato Rhythm

1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
◒	●	▲	●	◒	●	○	○	◒	●	▲	●	◒	●	○	○	◒	●	▲	●	◒	●	○	○
1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6
○	○	~	○	○	~	○	○	~	○	○	~	○	○	~	○	○	~	○	○	~	○	○	~
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24

Creating this diagram allowed a discussion about polyrhythms. The rhythms move through phases of being in sync (as at the counts of 1, 2, 3, 4, 5, 6 where the rhythms all result in a zero difference) and out of sync (as at counts like 12 where the rhythms creates a non-zero difference of -2.)

Here, the teachers had their first realization of familiar mathematics in the patterns. The rhythms re-aligned every 24 beats, which is the least common multiple of 8 and 6. The dancer's movements were the most calm at counts around the least common multiple and she appeared the most animated at counts further from 24. This happens because immediately following the least common multiple number of beats the rhythms are re-aligned at a count of "one", which is the point at which the dancer is (perhaps unknowingly) brought into the music and to some extent brought under the control of the drummer. Simply viewing the table of handstrokes in the polyrhythm caused a few of the teachers to ponder their prior knowledge. "Wow! I always figured there was a connection between music and math, but I never thought about it this way." And "I've watched some videos of African Dancing before and I it always seemed to me to be so random. But the math here makes me think it's more organized than I thought."

Finding the algebra

After the teachers analyzed the rhythms with basic arithmetic ideas like least common multiple, they looked more deeply at the two rhythms. Assuming one could judge the intensity of the dancer by simply watching, we noticed that her intensity seemed to remain constant for a few beats, then change to a different level (sometimes more intense and sometimes more calm.) It appeared that there were beats of the polyrhythm when the dancer's moves might be clustered into short bursts of energy.

Ordered pairs

The teachers decided to analyze the rhythms algebraically. They wanted to develop a general rule that might describe the rhythm. A good first step for them was to try and record the information into an *x*-*y* coordinate grid. To do that, they needed ordered pairs. The teachers created a list of ordered pairs where the counts of the polyrhythm: 1, 2, 3, …, 23, 24 served as the *x*-coordinates (the domain of the relation). The *difference* between the counts of the individual Tumbao and Bembe Ostinato rhythms served as the *y*-coordinate (the range). For instance, the 12^{th} count of the polyrhythm corresponds to the Tumabo at count #4 and the Bembe Ostinato at count #6. The difference between these two counts is $^{-}2$. So this ordered pair is (12, -2). The 24^{th} count of the polyrhythm occurs at Tumbao #8 and Bembe Ostinato # 6. In this way, the ordered pair: (24, 2) is created. Using this strategy, the teachers created an *x*-*y* table (Figure 3) to record the ordered pairs to describe the polyrhythm.

Figure 3

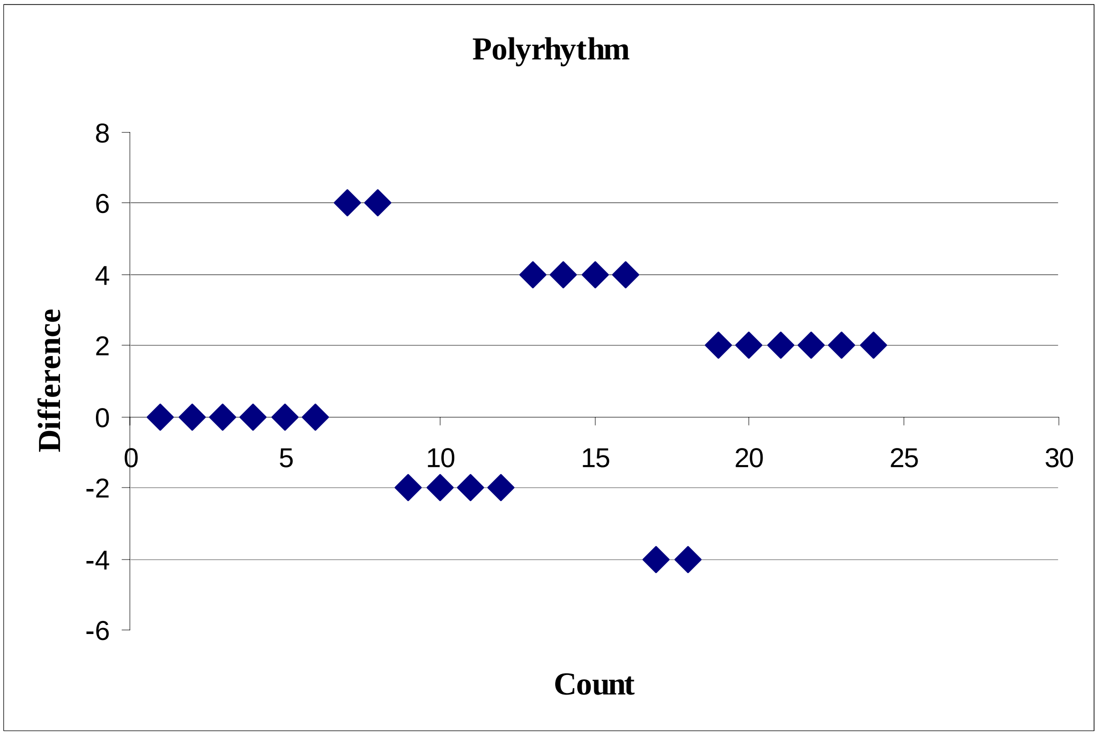
x	$T – BO$	Diff (y)
1	1 – 1	0
2	2 – 2	0
3	3 – 3	0
4	4 – 4	0
5	5 – 5	0
6	6 – 6	0
7	7 – 1	6
8	8 – 2	6
9	1 – 3	-2
10	2 – 4	-2
11	3 – 5	-2
12	4 – 6	-2
13	5 – 1	4
14	6 – 2	4
15	7 – 3	4
16	8 – 4	4
17	1 – 5	-4
18	2 – 6	-4
19	3 – 1	2
20	4 – 2	2
21	5 – 3	2
22	6 – 4	2
23	7 – 5	2
24	8 – 6	2

Written as ordered pairs, we have the relation: {(1,0), (2, 0), (3,0), (4,0), (5,0), (6,0), (7,6), (8, 6), (9, -2), (10, -2), (11, -2), (12, -2), (13, 4), (14, 4), (15, 4), (16, 4), (17, -4), (18, -4), (19, 2), (20, 2), (21, 2), (22, 2), (23,2), (24,2)}.

Domain & Range

Creating these ordered pairs led to discussions about domain and range, and discrete versus continuous graphs. Since the rhythms repeat continuously the domain for the relation is the set of all natural numbers, not just the numbers 1 – 24. However, the *y*-coordinates cycle through the same values every 24 counts. Therefore, the range consisted only of {0, 6, -2, 4, -4, 2}. A side discussion occurred and the teachers determined that the ordered pairs represent a discrete graph and the points should not be connected. Their graph is shown in Figure 4.

Figure 4 – A graph of the Polyrhythm



Function notation

Then, the teachers became curios to know whether or not the graph represented a relation that was a function. The vertical-line-test satisfied their inquiry. The relation is a function. They were then eager to represent the function with familiar and traditional algebra symbolism: $f(x)$. In order to determine an equation for the polyrhythm, they had to analyze the manner in which the coordinates were created. In particular, developing a symbolic representation for the repetitious nature of the *y*-coordinate proved to be quite a challenge. The first task was to determine which *x*-coordinate always matched up with any given *y*-coordinate. That is, which *y*-coordinate generated from the same counts in the polyrhythm. For instance, at *x* – coordinates 2, 26, 50, 74, … of the polyrhythm both individual rhythms were at counts of 2, so the *y*-coordinate would always be 0. At x – coordinates 10, 34, 58, 82, … ,the Tumbao is at count 2 and the Bembe Ostinato is at count 4. The y-coordinate would be – 2. (See Table 2). The repetitious nature of the y-coordinate was interesting to the teachers. After noting that the x-coordinates 1, 25, 49, 73, … always yielded a y-coordinate of 0, one teacher said, "The same thing is happening every 24 counts. That must be related to the least common multiple." Another teacher looked quite perplexed and said, "It seems like we have two functions here, somehow."

Transferring this information to more traditional and familiar functional notation ended up to be quite difficult. The teachers had to find a way to describe the seemingly-structured activity of the x-coordinate and to then match the correct cycle of the y-coordinates with those x-values. After involved discussions among the groups of teachers, they finally agreed to use the following notation: $x = 24a + c$, where a ranges through the values: {0, 1, 2, 3, …} and c ranges through the values: {1, 2, 3, …, 23, 24}. In this way, x generates the specific beats in the polyrhythm. For example, $x = 24(0) + 1$ is the 1st beat of the polyrhythm and $x = 24(1) + 1$ is the 25th beat of the polyrhythm (also marking the return of the two individual rhythms to the counts of 1 and 1). In general, then, if $x = 24a + 1$, where $a \in \{0, 1, 2, 3, 4, \ldots\}$, then $y = 0$. Further generalization of this approach allowed the teachers to determine the following representation for the function $f(x)$ of the polyrhythm.

$$f(x)=\begin{cases} 0 \text{ if } x=24a+c, \text{ where } a\in\{0,1,2,3,4,\ldots\} \text{ and } c\in\{1,2,3,4,5,6\} \\ 6 \text{ if } x=24a+c, \text{ where } a\in\{0,1,2,3,4,\ldots\} \text{ and } c\in\{7,8\} \\ -2 \text{ if } x=24a+c, \text{ where } a\in\{0,1,2,3,4,\ldots\} \text{ and } c\in\{9,10,11,12\} \\ 4 \text{ if } x=24a+c, \text{ where } a\in\{0,1,2,3,4,\ldots\} \text{ and } c\in\{13,14,15,16\} \\ -4 \text{ if } x=24a+c, \text{ where } a\in\{0,1,2,3,4,\ldots\} \text{ and } c\in\{17,18\} \\ 2 \text{ if } x=24a+c, \text{ where } a\in\{0,1,2,3,4,\ldots\} \text{ and } c\in\{19,20,21,22,23,24\} \end{cases}$$

Representing a function in this way is known as *piecewise*. One teacher said, “I have studied piecewise functions, but never really found them interesting or thought-provoking before.” Another said, “I don’t remember ever studying piecewise functions.” The teachers became interested in a mathematical understanding of what was happening to the dancer’s emotions for each piece of the function. They analyzed the function, determining that the *x* –coordinates for which the dancer seemed to exhibit the most dramatic movements occurred at *x* –coordinates {7, 8, 9, … 22, 23, 24} and the slower movements occurred at *x* coordinates { 1, 2, 3,4, 5, 6}. They recognized that she seemed to prepare to slow down at the *x*-coordinate of 24, the LCM.

Evaluation component

The main point of this lesson was to provide the teachers with a learning experience where they encountered algebra in an unexpected location. However, of equal importance was to demonstrate how the mathematics could be much more sophisticated than simple patterning. Too often, culturally-based mathematics lessons are vague and generally over simplified. After experiencing this lesson, the teachers were to create a set of culturally-based lessons for their algebra students. They selected a culture and studied it, looking for meaningful and rigorous algebraic connections. One teacher studied the artistry of Celtic knots, another studied the importance of phases of the moon in Norway, a third studied the importance of salmon to the

Native American Lummi, and a fourth studied the role of conch shells in ancient Aztec cultures of South Central Mexico. All of these teachers built algebra units from their studies.

Summary

Readers of this paper may feel overwhelmed as they think about the effort, time and energy we put into learning about African Drumming and developing it as an alternative context for algebra lessons. When asked to develop their lessons, the teachers often found themselves short on time. One teacher noted, "I cannot believe how much time it has taken me to really learn about the culture, let alone figure out the math." We all had to carefully manage other pressing responsibilities to develop these lessons from scratch. But if culturally-relevant mathematics lessons is our goal, then we have to make time to pursue true, deep knowledge. Finding a colleague who may serve as an expert for the cultural and/or mathematical components was a good place for us to start. We encourage the readership to look for unexpected contexts that may provide an avenue for teacher educators to provide some measure of social justice. We offer not only the African Drumming information that we learned, but also suggest that our teachers' units also provide some evidence of a successful effort.

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